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Section 3: Introduction to Functions

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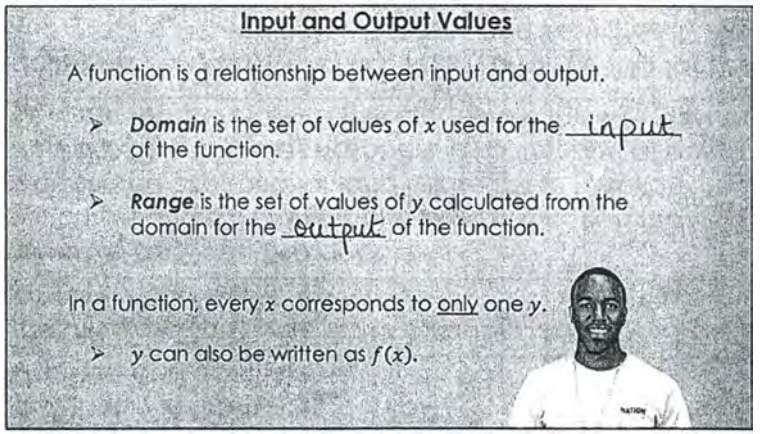


The following Mathematics Florida Standards will be covered in this section:
A-APR.1.1 - Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
A-SSE.1.1 a - Interpret expressions that represent a quantity in terms of its context. a. Interpret parts of an expression, such as terms, factors, and coefficients.
A-SSE.1.2 - Use the structure of an expression to identify ways to rewrite it. *For example,* see $x^4 - y^4$ *as* $(x^2)^2 - (y^2)^2$ *thus recognizing it as a difference of squares that can be factored as* $(x^2 - y^2)(x^2 + y^2)$.
F-BF.1.1 be - Write a function that describes a relationship between two quantities. b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by* adding *a constant function to a decaying exponential,* and *relate these functions to the model.* c. Compose functions. *For example, if T(y) is the temperature in the atmosphere as a function of height,* and *h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.*
F-BF.2.3 - Identify the effect on the graph of replacing *f (x)* by *f (x) + k, k f(x), f (kx),* and *f (x + k)* for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing* even and odd *functions from their graphs* and *algebraic expressions for them.*
F-IF.1.1 - Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and xis an element of its domain, then *f(x)* denotes the output *of f* corresponding to the input *x.* The graph *of f* is the graph of the equation y = *f(x).*
F-IF.1.2 - Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
F-IF.2.4 - For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums* and *minimums; symmetries; end behavior;* and *periodicity.*
F-IF.2.5 - Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*
F-IF.2.6 - Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Section 3: Introduction to Functions

Section 3-Topic 1
Input and Output Values

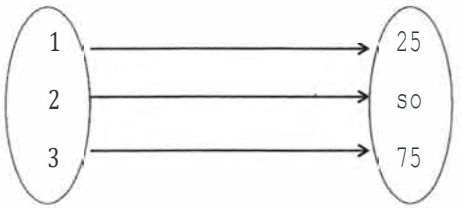
A function is a relationship between input and output.

- ***Domain*** is the set of values of x used for the _____ of the function.
- ***Range*** is the set of values of y calculated from the domain for the ____ of the function.

In a function, every x corresponds to only one y.

- y can also be written as f ***(x).***

Consider the following function.



For every x (input domain) there is a unique y (output range).

We also refer to the variables as independent and dependent. The dependent variable _____ _ ____ the independent variable.

Refer to the mapping diagram on the previous page.

Which variable is independent?

Which variable is dependent?

Consider a square whose perimeter depends on the length of its sides.

What is the independent variable?

What is the dependent variable?

How can you represent this situation using function notation?

STUDY EDGE TIP

We can choose any letter to represent a function, such as f *(x)* or ***g(x),*** where x is the input value. By using different letters, we show that we are talking about different functions.

Let's Practice!

1. You earn $10.00 per hour babysitting. Your total earnings depend on the number of hours you spend babysitting.

   a. What is the independent variable?

   b. What is the dependent variable?

   c. How would you represent this situation using function notation?

2. The table below represents a relation.

x	y
-3	5
0	4
2	6
-3	8

   a. Is the relation also a function? Justify your answer.

   b. If the relation is not a function, what number could be changed to make it a function?

Try It!

3. Mrs. Krabappel is buying composition books for her classroom. Each composition book costs $1.25.

   a. What does her total cost depend upon?

   b. What are the input and output?

   c. Write a function to describe the situation.

   d. If Mrs. Krabappel buys 24 composition books, they will cost her $30.00. Write this function using function notation.

4. Consider the following incomplete mapping diagrams.

Diagram A

Diagram B

Diagram C

a. Complete Diagram A so that it is a function.

b. Complete Diagram B so that it is NOT a function.

c. Is it possible to complete the mapping diagram for Diagram C so it represents a function? If so, complete the diagram to show a function. If not, justify your reasoning.

BEAT THE TEST!

1. Isaac Messi is disorganized. To encourage Isaac to be more organized, his father promised to give him three dollars for every day that his room is clean and his schoolwork is organized.

Part A: Define the input and output for the given scenario.

Input:

Output:

Part B: Write a function to represent this situation.

2. The cost to manufacture x pairs of shoes can be represented by the function $C(x) = 63x$. Complete the statement about the function.

If C(6) = 378, then | o / 6 / 63 / 378 | pairs of shoes cost | $6. / $189. / $378. / $2,268.

3. Which of the following relations is not a function?

@ {(0,5),(2,3),(5,8),(3,8)}
® {(4, 2), (-4, 5), (0, 0)}
© {(6,5),(4,1),(-3,2),(4,2)}
@ {(-3, -3), (2, 1), (5, -2)}

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Section 3 - Topic 2
Representing, Naming, and Evaluating Functions

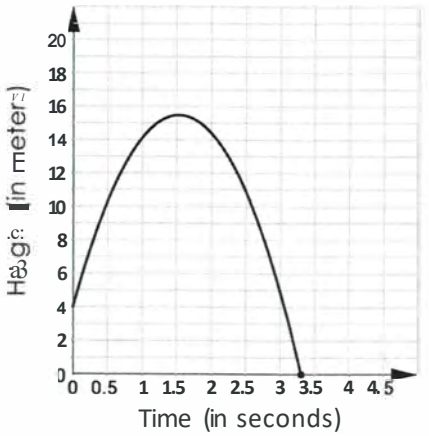
A ball is thrown into the air with an initial velocity of 15 meters per second. The quadratic function $h(t) = -4.9t^2 + 15t + 4$ represents the height of the ball above the ground, in meters, with respect to time t, in seconds.

Determine h(2) and explain what it represents.

Would -3 be a reasonable input for the function?

The graph below represents the height of the ball with respect to time.

Height of the Ball Over Time



What is a reasonable domain for the function?

What is a reasonable range for the function?

Let's Practice!

1. On the moon, the time, in seconds, it takes for an object to fall a distance, *d*, in feet, is given by the function $f(d) = 1.11\sqrt{d}$.

   a. Determine $f(5)$ and explain what it represents.

   b. The South Pole-Aitken basin on the moon is 42,768 feet deep. Determine a reasonable domain for a rock dropped from the rim of the basin.

2. Floyd drinks two Mountain Dew sodas in the morning. The function that represents the amount of caffeine, in milligrams, remaining in his body after drinking the sodas is given by $f(t) = 110(0.8855)^t$ where t is time in hours. Floyd says that in two days the caffeine is completely out of his system. Do you agree? Justify your answer.

Try It!

3. Medical professionals say that 98.6°F is the normal body temperature of an average person. Healthy individuals' temperatures should not vary more than 0.5°F from that temperature.

   a. Write an absolute value function $f(t)$ to describe an individual's variance from normal body temperature, where t is the individual's current temperature.

   b. Determine $f(101.5)$ and describe what that tells you about the individual.

   c. What is a reasonable domain for a healthy individual?

BEAT THE TEST!

1. The length of a shipping box is two inches longer than the width and four times the height.

   Part A: Write a function $V(w)$ that models the volume of the box, where w is the width, in inches.

   Part B: Evaluate $V(10)$. Describe what this tells you about the box.

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Section 3 - Topic 3
Adding and Subtracting Functions

Let $h(x) = 2x^2 + x - 5$ and $g(x) = -3x^2 + 4x + 1$.

Find $h(x) + g(x)$.

Find $h(x) - g(x)$.

Let's Practice!

1. Consider the following functions.

$$f(x) = 3x^2 + x + 2$$
$$g(x) = 4x^2 + 2(3x - 4)$$
$$h(x) = 5(x^2 - 1)$$

a. Find $f(x) - g(x)$.

b. Find $g(x) - h(x)$.

. Try It!

2. Recall the functions we used earlier.

$$f(x) = 3x^2 + x + 2$$
$$g(x) = 4x^2 + 2(3x - 4)$$
$$h(x) = 5(x^2 - 1)$$

a. Let $m(x)$ be $f(x) + g(x)$. Find $m(x)$.

b. Find $h(x) - m(x)$.

BEAT THE TEST!

1. Consider the functions below.

$$f(x) = 2x^2 + 3x - 5$$
$$g(x) = 5x^2 + 4x - 1$$

Which of the following is the resulting polynomial when $f(x)$ is subtracted from $g(x)$?

® $-3x^2 - x - 4$
® $-3x^2 + 7x - 6$
(S) $3x^2 + x + 4$
@ $3x^2 + 7x - 6$

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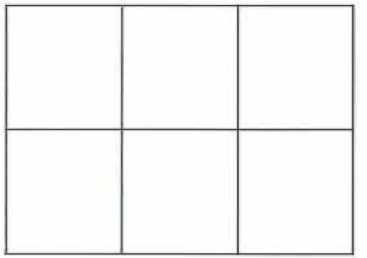
Section 3-Topic 4
Multiplying Functions

Use the distributive property and modeling to perform the following function operations.

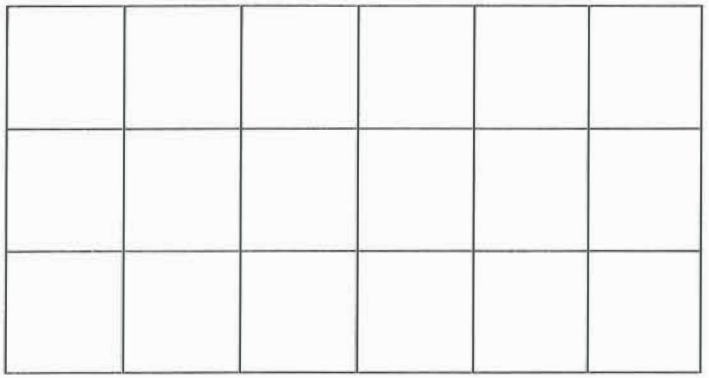
Let $f(x) = 3x^2 + 4x + 2$ and $g(x) = 2x + 3$.

Find $f(x) \cdot g(x)$.



Let $m(y) = 3y^5 - 2y^2 + 8$ and $p(y) = y^2 - 2$.

Find $m(y) \cdot p(y)$.



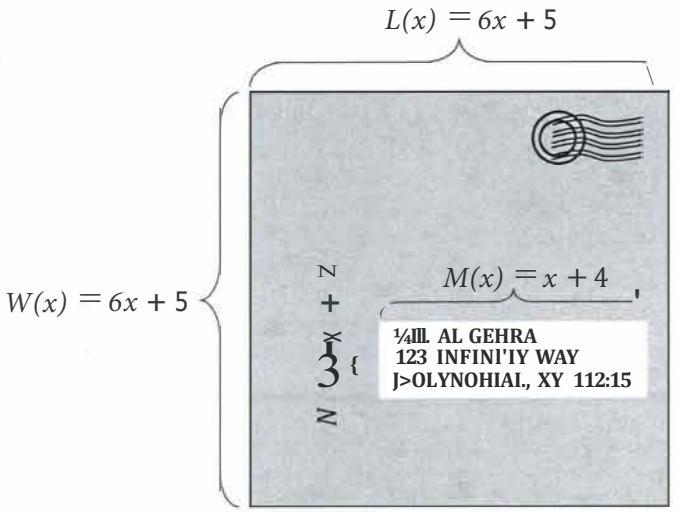
Let's Practice!

1. Let $h(x) = x - 1$ and $g(x) = x^3 + 6x^2 - 5$.

   Find $h(x) \cdot g(x)$.

Try It!

2. The envelope below has a mailing label.



a. Let $A(x) = L(x) \cdot W(x) - M(x) \cdot N(x)$. Find $A(x)$.

b. What does the function $A(x)$ represent in this problem?

BEAT THE TEST!

1. The length of the sides of a square ares inches long. A rectangle is six inches shorter and eight inches wider than the square.

   Part A: Express both the length and the width of the rectangle as a function of a side of the square.

   Part B: Write a function to represent the area of the rectangle in terms of the sides of the square.

2. Felicia needs to find the area of a rectangular field in her backyard. The length is represented by the function $L(x) = 4x^4 - 3x^2 + 6$ and the width is represented by the function $W(x) = x + 1$. Which of the following statements is correct about the area, $A(x)$, of the rectangular field in Felicia's backyard? Select all that apply.

   0 $A(x) = 2[L(x) + W(x)]$
   D The resulting expression for $A(x)$ is a fifth-degree polynomial.
   D The resulting expression for $A(x)$ is a polynomial with a leading coefficient of 5.
   0 The resulting expression for $A(x)$ is a binomial with a constant of 6.
   0 $W(x) = \frac{A(x)}{L(x)}$

Section 3 - Topic 5
Closure Property

When we add two integers, what type of number is the sum?

When we multiply two irrational numbers, what type of numbers could the resulting product be?

A set is _____ for a specific operation if and only if the operation on two elements of the set ***always*** produces an element of the same set.

Are integers closed under addition? Justify your answer.

Are irrational numbers closed under multiplication? Justify your answer.

Are integers closed under division? Justify your answer.

Let's apply the closure property to polynomials.

Are the following statements true or false? If false, give a counterexample.

Polynomials are closed under addition.

Polynomials are closed under subtraction.

Polynomials are closed under multiplication.

Polynomials are closed under division.

Let's Practice!

1. Check the boxes for the following sets that are closed under the given operations.

Set	+	−	×	÷
{0, 1, 2, 3, 4, ... }	D	D	D	D
{... , - 4 , - 3 , - 2 , - 1 }	D	D	D	D
{... , - 3 , - 2 , - 1 , 0 , 1, 2, 3, ... }	D	D	D	D
{rational numbers}	D	D	D	D
{polynomials}	D	D	D	D

Try It!

2. Ms. Sanabria claims that the closure properties for polynomials are analogous to the closure properties for integers. Mr. Roberts claims that the closure properties for polynomials are analogous to the closure properties for rational numbers. Who is correct? Explain your answer.

BEAT THE TEST!

1. Choose from the following words and expressions to complete the statement below.

$2x^5 + (-3y^2) - 2$ | $(5y)^2 + 4x + 3y^3$ | $5y - 1 + 7x^2 + 8y^2$

integers | ~~variables~~ | whole numbers

coefficients | rational numbers | exponents

The product of $5x^4 \cdot 3x^2 + 2$ and _____ illustrates the closure property because the _____ of the product are _____ and the product is a polynomial.

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Section 3 - Topic 6
Real-World Combinations and Compositions of Functions

There are many times in real world situations when we must combine functions. Profit and revenue functions are a great example of this.

Let's Practice!

1. At the fall festival, the senior class sponsors hayrides to raise money for the senior trip. The ticket price is $5.00 and each hayride carries an average of 15 people. They consider raising the ticket price in order to earn more money. For each $0.50 increase in price, an average of 2 fewer seats will be sold. Let x represent the number of $0.50 increases.

   a. Write a function, $T(x)$, to represent the cost of one ticket based on the number of increases.

   b. Write a function, $R(x)$, to represent the number of riders based on the number of increases.

   c. Write a revenue function for the hayride that could be used to maximize revenue.

Try It!

2. The freshman class is selling t-shirts to raise money for a field trip. The cost of each custom designed t-shirt is $8. There is a $45 fee to create the design. The class plans to sell the shirts for $12.

   a. Define the variable.

   b. Write a cost function.

   c. Write a revenue function.

   d. Write a profit function.

Let's Practice!

3. Priscilla works at a cosmetics store. She receives a weekly salary of $350 and is paid a 3% commission on weekly sales over $1500.

   a. Let x represent Priscilla's weekly sales. Write a function, $f(x)$, to represent Priscilla's weekly sales over $1500.

   b. Let x represent the weekly sales on which Priscilla earns commission. Write a function, $g(x)$, to represent Priscilla's commission.

   c. Write a composite function, $(g \circ f)(x)$ to represent the amount of money Priscilla earns on commission.

Try It!

4. A landscaping company installed a sprinkler that rotates and sprays water in a circular pattern. The water reaches its maximum radius of 10 feet after 30 seconds. The company wants to know the area that the sprinkler is covering at any given time after the sprinkler is turned on.

   a. Let t represent the time in seconds after the sprinkler is turned on. Write a function, $r(t)$, to represent the size of the growing radius based on time after the sprinkler is turned on.

   b. Let r represent the size of the radius at any given time. Write a function, $A(r)$, to represent the area that the sprinkler covers at any given time, in seconds.

   c. Write a composite function, $A(r(t))$ to represent the area based on the time, in seconds, after the sprinkler is turned on.

BEAT THE TEST!

1. A furniture store charges 6.5% sales tax on the cost of the furniture and a $20 delivery fee. (The delivery fee is not subject to sales tax.)

   The following functions represent the situation:

   $$f(a) = 1.065a$$
   $$g(b) = b + 20$$

   Part A: Write the function $g(f(a))$.

   Part B: Match each of the following to what they represent. Some letters will be used twice.

a	A. The cost of the furniture, sales tax, and delivery fee.
b	B. The cost of the furniture and sales tax.
$f(a)$	C. The cost of the furniture.
$g(b)$	
$g(f(a))$	